

# **Adaptive Multigrid Hurricane Modeling**

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## **LONG-TERM GOALS**

To facilitate better prediction of tropical cyclones by: (1) developing numerical methods with improved accuracy and efficiency, (2) investigating the sensitivity of tropical cyclone motion to vortex structure and model resolution, and (3) increasing understanding of the potential vorticity mixing process in hurricanes.

## **OBJECTIVES**

This work focuses on the development, testing, and application of an adaptive multigrid barotropic tropical cyclone track model (MUDBAR). The specific objectives of the current effort are to:

1. Develop adaptive multigrid techniques based on truncation error estimates,
2. Implement higher-order discretization via compact differencing and extrapolation,
3. Extend adaptive multigrid techniques to the shallow-water model,
4. Use high-resolution numerical experiments to study the sensitivity of tropical cyclone tracks to differences in vortex structure and model resolution, and
5. Use the MUDBAR model to verify and extend previous studies of potential vorticity (PV) mixing in hurricanes.

## **APPROACH**

Adaptive multigrid techniques offer the potential of fast computation and high accuracy; they differ from conventional nested-grid methods by making full use of the interplay between discretizations on the different grids. Their potential has been demonstrated in a multigrid barotropic hurricane track model (MUDBAR) developed by the PI, which runs ten to twenty times faster than a uniform-resolution version for the same level of accuracy. A preliminary version of this model using fixed-size movable grids was described in Fulton (1997). Our current work involves development and testing of numerical methods, numerical experiments with the model, and complementary analytical studies. Project personnel for this year include the PI and three graduate students (Nicole Burgess, Brittany Mitchell, and Hu Miao).

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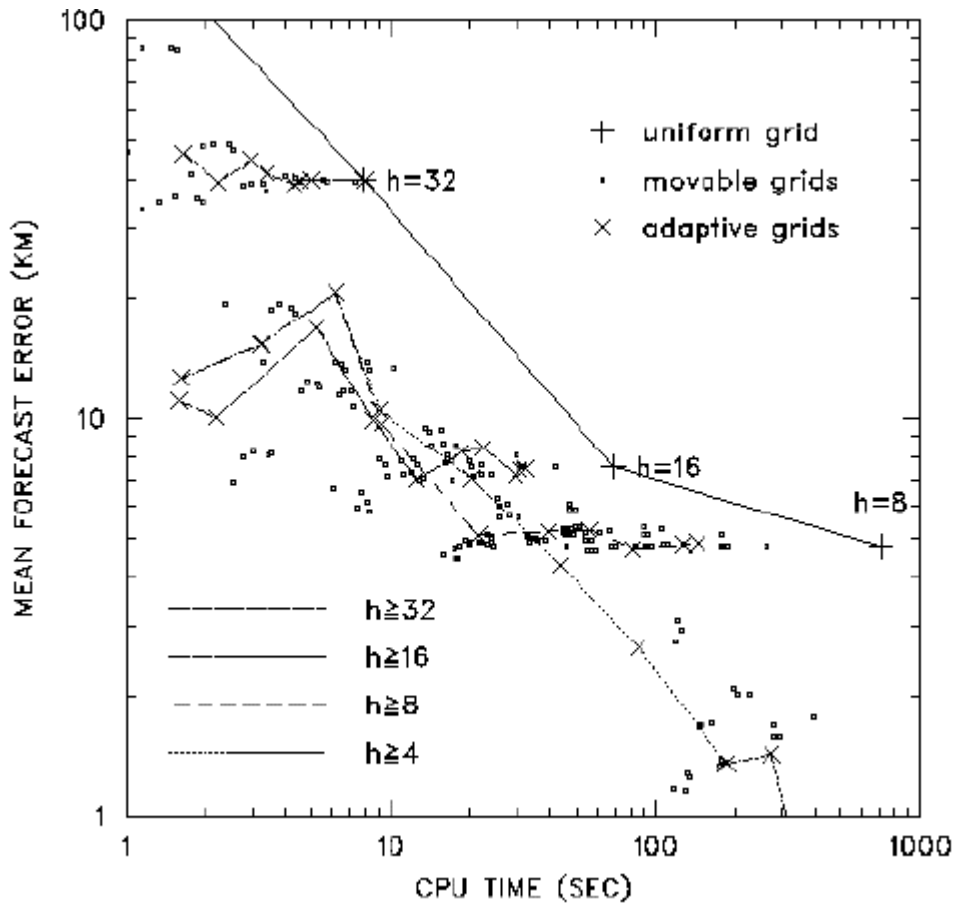
## WORK COMPLETED

1. Implementation of adaptive mesh refinement in the MUDBAR model is complete. Grids are now chosen automatically, based on truncation error estimates, resulting in a seamless discretization that nearly optimizes the trade-off between accuracy and efficiency. A paper describing the adaptive multigrid method and its performance has been submitted to *Monthly Weather Review* (Fulton, 1999).
2. Higher-order multigrid methods based on compact differencing and extrapolation methods have been studied and compared. The M.S. project of Nicole Burgess (Burgess and Fulton, 1999) showed that compact differencing produces higher accuracy than tau-extrapolation; subsequent work has demonstrated an algorithm which solves to the level of truncation error in a fixed number of multigrid cycles, independent of the mesh size. The fourth-order method has been incorporated into the MUDBAR model for the streamfunction equation; however, the overall improvement in accuracy so far is small, since the vorticity equation currently uses a second-order discretization for the Jacobian.
3. We have investigated the possibility of finding a compact fourth-order discretization of the Jacobian which conserves vorticity, enstrophy, and kinetic energy, similar to the second-order Arakawa Jacobian. Our analysis shows that while a one-parameter family of conservative second-order Jacobians exists, there is no such fourth-order Jacobian which is compact (i.e., using only the nearest neighboring points). Thus, completing the fourth-order version of MUDBAR will require noncompact differencing or extrapolation; this is the current task of graduate student Hu Miao.
4. A shallow-water version of MUDBAR has been developed and is currently being tested. This is the M.S. thesis work of graduate student Brittany Mitchell, and is expected to be complete by the end of the year.
5. Numerical experiments of PV mixing using MUDBAR have been conducted to verify and extend the earlier results of Schubert et al. (1999).
6. The minimum enstrophy solution for the shallow water model derived last year has been solved numerically and compared with the corresponding solution for the nondivergent barotropic model. Preliminary results were reported in Fulton et al. (1999).

## RESULTS

The principal technical results achieved through this work in FY99 are:

- The adaptive multigrid method has been shown to be a practical and robust technique for achieving nonuniform resolution for nonlinear flow problems. Figure 1 shows an example of the accuracy and efficiency obtained with the adaptive multigrid method in comparison to using movable grids (of fixed size) or uniform grids. The adaptive multigrid method automatically chooses reasonable grids with minimal human intervention; it can be “tuned” by a single parameter (the exchange rate) to specify the desired tradeoff between increased accuracy and increased computational work. The method makes it possible to resolve the details of the vortex at small scales while representing the surrounding large-scale flow efficiently.



**Figure 1: Accuracy and efficiency of MUDBAR model**

- Using numerical experiments of PV mixing with the MUDBAR model, we have confirmed and extended the earlier results of Schubert et al. (1999). In particular, we have shown that the characteristic wavenumber-four instability found with an unstable barotropic vortex at rest in a doubly-periodic spectral model on the  $f$ -plane persists when the vortex is embedded in a zonal flow on the beta-plane or sphere. Thus, this qualitative behavior of the chaotic nonlinear PV mixing process is indeed a feature of the dynamics, rather than an artifact of the assumption of periodicity. Figure 2 shows the vortex structure for the PV ring experiment of Schubert et al. (1999) as run in the MUDBAR model. Only a small portion of the 4096 km square domain is shown, with solid lines indicating the boundaries of three of the six grid patches. This experiment used mesh spacings ranging from 0.5 km in the inner region to 32 km in the environment, thus representing both the fine-scale details of the PV mixing and the large-scale environmental flow both accurately and efficiently. For this case the adaptive model was six times faster than a movable-mesh version of similar accuracy, and orders of magnitude faster than using a single uniform grid.
- Comparisons of the minimum enstrophy solutions for the shallow water and nondivergent barotropic models show only slight differences; thus, conclusions based on studies of PV mixing in the latter model may have validity beyond that simple context.



## TRANSITIONS

The MUDBAR model has not yet been used by others researchers; any opportunities for collaboration in model testing, implementation, or intercomparisons would be welcomed. Numerical techniques being developed for solving the minimum enstrophy problem will likely be applied by members of Wayne Schubert's group for minimum enstrophy studies on the sphere.

## RELATED PROJECTS

The PI received a DURIP grant (N00014-98-1-0368) which has provided funds for computer equipment used in this research.

A group at Colorado State University headed by Wayne Schubert and Mike Montgomery has been studying the potential vorticity mixing process. Work on this subject reported here has been done in close collaboration with this group.

## REFERENCES

Fulton, S. R., 1997: A comparison of multilevel adaptive methods for hurricane track prediction. *Elec. Trans. Numer. Anal.*, **6**, 120-132. (<http://etna.mcs.kent.edu>)

## PUBLICATIONS

Fulton, 1999: An adaptive multigrid barotropic tropical cyclone track model. Submitted to *Mon. Wea. Rev.* Preprint available from <http://www.clarkson.edu/~fulton/pubs>.

Schubert, W. H., M. T. Montgomery, R. K. Taft, T. A. Guinn, S. R. Fulton, J. P. Kossin, and J. P. Edwards, 1999: Polygonal eyewalls, asymmetric eye contraction, and potential vorticity mixing in hurricanes. *J. Atmos. Sci.*, **56**, 1197-1223.

### Non-refereed publications and preprints:

Burgess, N. M., and S. R. Fulton, 1999: Experiments with higher-order multigrid methods. Technical Report No. 99-02, Department of Mathematics and Computer Science, Clarkson University. Available from <http://www.clarkson.edu/~mcs/reports.html>.

Fulton, S. R., N. M. Burgess, and B. L. Mitchell, 1999: Experiments with a self-adaptive multigrid barotropic tropical cyclone track model. Preprints, 23<sup>rd</sup> AMS Conference on Hurricanes and Tropical Meteorology, 459-460.

Fulton, S. R., W. H. Schubert, and M. T. Montgomery, 1999: Potential vorticity mixing in hurricanes: comparison of nondivergent and divergent barotropic vortices. Preprints, 23<sup>rd</sup> AMS Conference on Hurricanes and Tropical Meteorology, 686-688.

Schubert, W. H., M. T. Montgomery, S. R. Fulton, J. P. Kossin, and R. K. Taft, 1999: Potential vorticity mixing in hurricanes: an overview. Preprints, 23<sup>rd</sup> AMS Conference on Hurricanes and Tropical Meteorology, 676-677.